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### Abstract

An improvement in the evaluation of the Cs beam primary frequency standard NBS-6 is being attempted through a reevaluation of Rabi pulling using a recently published theory. Time of flight distribution measurements and frequency measurements at various C-field values have been performed in both beam directions. This allows us to model Rabi pulling and hence more clearly study other systematic effects.

Results presented here show the relative magnitude of the two effects (Rabi pulling and cavity phase shift). Zero crossings of Rabi pulling are identified and demonstrated to be near the C-field settings which give minimum power dependence. The results are preliminary in the sense that full evaluation of Rabi pulling should include analysis of the effect in the two beam directions as a function of source and detector position, in order to better guarantee configuration repeatability. It is expected that total evaluation accuracy of NBS-6 might improve to a few times 10<sup>-14</sup> once the indicated experiments are properly performed and analyzed.

# Introduction

The end-to-end cavity phase shift is among the major uncertainty sources in primary Cs beam frequency standards. It comes from the residual asymmetry existing in the most carefully built Ramsey cavity and its value is given by the formula [1]:

$$\frac{\delta v}{v} = \frac{\ell}{2\pi L} \cdot \frac{\langle \delta_{\phi}(\tau) \rangle}{\langle \tau \rangle}$$
 (1)

where L and  $\ell$  are the lengths of Ramsey and Rabi cavities, as shown in fig. 1,  $\tau$  is the time of flight (TOF) through a single Rabi cavity for a given velocity group and  $\delta_{\phi}(\tau)$  is the average phase difference of the microwave interrogating field experienced by that group between first and second cavity. The size of this effect can be several  $10^{-13}$  in well designed long cavities [2].

Various techniques have been proposed to avoid or minimize cavity phase shift, ranging from the use of superconducting cavities [3] to the dual frequency technique [4]. However the most immediate approach to the problem is still to measure the effect. Several methods have been devised for this purpose. All are based on varying in a controlled way one of the accessible variables affecting its value in (1), namely  $\langle \tau \rangle$  or (the sign of )  $\langle \delta_{\alpha}(\tau) \rangle$ .

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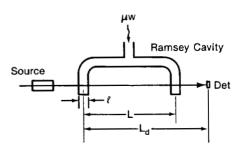


Fig. 1. Schematic and geometrical definitions in an atomic beam with a Ramsey cavity. In NBS-6  $\ell=1$ cm; L=3.75m; and L<sub>d</sub>=4.54m.

In the power shift method the effective  $\langle \tau \rangle$  is changed by the microwave power velocity selection, based on the fact that the transition probability depends on  $\tau$ . Averaging over  $\tau$  is necessary due to the large TCF window. This method is best applied if coupled with beam reversal [5]. In the pulse method [6,7] the same effect is obtained more directly by introducing a very narrow TOF window with a strobed microwave power, based on the fact that atoms which are excited in both cavities have twice the transition probability of those which are excited in one only.

In the beam reversal method the order of the cavities is changed, which gives in principle a switch in sign for  $\delta_{\varphi}$  and a frequency value symmetrically shifted from the unperturbed frequency.

Unfortunately all these experiments are not clean in the sense that other effects of the same order of magnitude may well come into the picture with an intrinsic connection and affect the results. In particular this is true of distributed cavity phase shift [8,9,10] if the beam does not skim the cavity ends, and of Rabi pulling [11]. Both these effects can affect the two methods based on variations of  $\langle \tau \rangle$ , through the velocity dependence of trajectories (and hence of  $\delta_{\phi}$ ) and of the shape of Rabi wings; the power shift method in addition is difficult to interpret because of the dependence of Rabi pulling on microwave power [11], unless used near a zero crossing of the latter. Rabi pulling perturbs also the beam reversal method, due to the fact that TOF distributions are not the same in the two beam directions and may therefore cause different pulling even at the same C-field.

In this paper we report on a beam reversal experiment on the NBS-6 primary cesium standard in which the frequency was measured in both beam directions at various settings of the C-field in order to separate the effect of Rabi pulling from that of cavity phase shift. The latter in fact is expected not to depend on C-field.

TOF distribution measurements were performed in both beam directions and corrected for the transfer function of the detector system in order to compare calculated Rabi pulling curves to experimental results. The purpose of this experiment was to obtain high accuracy in the evaluation of phase shift through best fit of theoretical curves and experimental points to vield high accuracy in positioning the null Rabi pulling line, and to gain information about the zero crossings of Rabi pulling. The best operating points, where the power dependence of all residual effects compensate to yield insensitivity to microwave power, are expected to be found near the C-field settings for which the zero crossings occur. Only  $\Delta m = 0$  transitions were considered in the analysis, however it appears that intermediate  $\Delta m = \pm 1$  transitions may be just as important for Rabi pulling despite their small size, since they are nearer to the central clock transition.

### TOF Distributions

Evaluation of TOF distributions in states m = 0,  $\pm$  1 was carried out for both beam directions with a time of flight method using  $L_d$  as a length basis. Microwave pulses 1.3 ms long were used at a 90 ms repetition rate and the detected signal was then processed in a multichannel signal averager for various microwave power levels for each of the analyzed m sublevels. In fig. 2 the envelopes of these responses are reported for the three central sublevels in the west-to-east beam direction (WE). First and second peaks come from the second and first cavity.

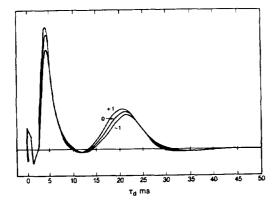


Fig. 2. Envelopes of detected pulse responses at various microwave power levels for the three central  $\Delta m=0$  transitions ( $m=0,\pm 1$ ). The small feature on the left is leakage into the detector of the microwave pulsing signal.

The waveform distortion effect of the detector system's transfer function is evident in the non ideal ratio between the heights and delays from the microwave pulse of the two peaks. Ideally this ratio should be  $\xi = (1-L/L_d)^{-1} = 5.75$  as can be seen from Fig. 1, but the limited bandwidth of the system tends to cut the sharp first peak and alter the relative positioning of the two by introducing a delay which is more important for the sharper peak. Unfortunately the transfer function cannot be easily measured because it incorporates parts of the detection system which are not accessible for controlled excitation from the outside of the tube. A computer program was assembled to perform the

deconvolution from the detector system's transfer function. An input waveform  $\mathbf{x}(\mathbf{t_d})$  of the form

$$x(\tau_d) = \xi \rho(\xi \tau_d) + \rho(\tau_d)$$
 (2)

where  $\rho(\tau_d)$  is the unknown TOF distribution, was assumed to be the beam response to the microwave pulse, and the calculated output waveform from the assumed detector system was compared to the experimental waveform. Optimization of a smoothed least square fit for this comparison for different zero-pole configurations of the detector system yielded consistently the same detector transfer function for all measured waveforms in both directions, and therefore a high degree of confidence in the capability of the computer program to find real TOF distributions. In fig. 3 the deconvolved distributions for the m = 0 level

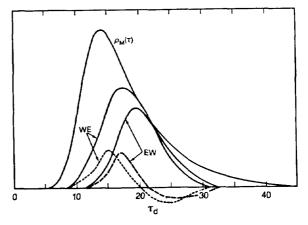


Fig. 3. Real TOF distributions  $\rho_O(\tau_d)$  for the two beam directions (solid lines) obtained by deconvolution from the detector system's transfer function are compared to the Maxwellian distribution  $\rho_M(\tau_d)$ . The broken lines are distributions  $\Delta_1(\tau_d)$  as obtained using (3).

in both directions are reported. Comparison with the Maxwellian distribution in the beam, calculated for the oven temperature of  $79\,^{\circ}\text{C}$  used in the experiments, enables one to recover the relative heights of the two distributions (consistent with measured beam intensities) and to calculate the beam optics selection functions  $G_{0}(\tau_{d})$  for the two beam directions, which are shown in Fig. 4. In fig. 3

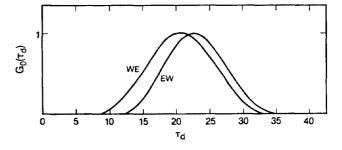


Fig. 4. Beam optics appear different in the two beam directions. TOF windows for the m=0 sublevel as derived from Fig. 3 are shown here.

the differential distributions  $\Delta_1(\tau_d) = \rho_1(\tau_d) - \rho_{-1}(\tau_d)$  are also shown for the two directions. Because of the more acceptable signal-to-noise ratio in the acquisition of  $\rho_0$  with respect to the difference  $\Delta$ , the latter was calculated from the approximated formula

$$\Delta_{1}(\tau_{d}) = \varepsilon_{1}\tau_{d} \frac{dG_{o}}{d\tau_{d}} \rho_{M}(\tau_{d})$$
 (3)

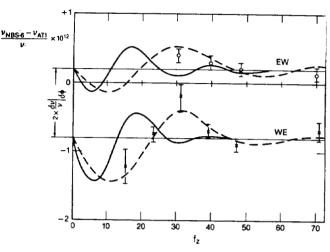
in which  $\epsilon_1$  was evaluated from the asymmetry in the  $\pm$ 1 transitions saturated intensity [11] using the corrected distributions for evaluation of  $\langle\tau\rangle$ , and from the relative displacements of calculated  $G_{\pm1}(\tau_d)$  with respect to  $G_0(\tau_d)$ . It turned out that all data were consistent if  $\epsilon_1$  was assumed to be 8% in both directions, which corresponds to an average field of 0.4 T in the deflecting magnets. This value is consistent with the published data of 0.9 T at the tip of the convex pole-piece [12]. The  $\Delta$  distributions calculated in this way and reported in Fig. 3 were therefore used for the calculation of the Rabi-pulling effect by the formulas given in [11].

### Frequency Measurements

Averaging times of 6 to 12 hours were used in frequency measurements, which should in principle provide an uncertainty of few parts in  $10^{14}\,$  from the NBS-6 stability contribution at the low source temperature used in the experiment. The measurements were taken by comparison with SPHM-4 passive hydrogen maser, which is stable in the  $10^{-14}\,$  region for that length of time [13], and then referred on a two day basis to the frequency of AT1, which is constructed from an average of commercial standards and is stable in the low  $10^{-14}\,$  from two days on. The 10 measurement uncertainty was estimated to be  $\pm$  5 x  $10^{-14}\,$ , at least for the points with small microwave power dependence.

Two series of measurements were taken in the two beam directions at various C-field and microwave power settings; the results for optimum microwave power are reported as a function of the Zeeman frequency of the m = 1 component in fig. 5. Unfortunately the time available for these measurements did not allow us to complete the set at low C fields for the EW direction, it is however evident that the general behavior of the experimental points agrees with the shape of the theoretically calculated curves.

The power dependence of a point near a zero crossing of Rabi pulling and near a maximum are shown in fig. 6 for the WE beam direction.



Zeeman frequency for the m=1 transition (kHz)

Fig. 5. Frequency measurements, corrected for C-field shift, are reported here for the two directions (O=EW, x=WE) as a function of the Zeeman frequency of the m=±1  $\Delta$ m=0 transition. The solid curves are Rabi-pulling curves calculated from the TOF distributions of Fig. 3. It appears that a stretching of these curves by a factor of approximately 2 is necessary to match experimental points. It is suggested that the spurious  $\Delta$ m=±1 transitions may have an important role in this disagreement.

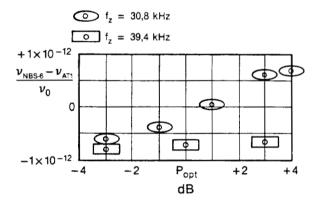


Fig. 6. Microwave power dependence of frequency for two C-field settings in the WE direction. The standard shows a much lower power dependence near the zero crossings of Rabi pulling.

# Effect of the $\Delta m = \pm 1$ Transitions

In fig. 5 the full curves are the result of calculations based on the measured TOF distributions, corrected for the detector system's transfer function. It turns out that a stretching of these curves (shrinking of the Zeeman axis) by about a factor of 2 would provide a very good match of theoretical curves and experimental points. The broken curves in fig. 5 have been obtained by stretching the solid curves by a factor of 1.85.

This fact seems to suggest that the  $\Delta m$  =  $\pm$  1 field dependent transitions, which are in principle small because they are excited by the unwanted oscillating B field component perpendicu-

lar to C-field, but are twice as near to the (0,0) transition than the (1,1) and (-1,-1), may have a strong effect due to the differential slope of their wings.

This hypothesis seems surprising if the size of these transitions is observed at a high C-field at which they can be isolated, as in fig. 7. However an actual comparison of measured lineshapes shows in fig. 8 that it may well be the case that these  $\Delta m = \pm 1$  transitions have a greater pulling effect than the neighboring  $\Delta m = 0$  transitions. Further analysis and experimental data are clearly necessary to realize an accurate description of this aspect of the problem.

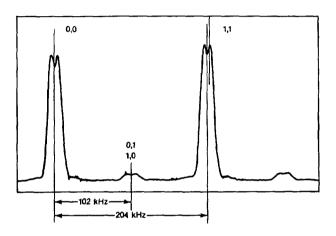


Fig. 7. Tube response at high C-field. Power is about 1 dB above optimum.

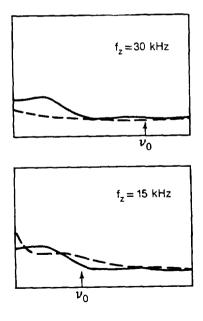


Fig. 8. Comparison of the wings of a  $\Delta m=\pm 1$  transition (solid line) and a  $\Delta m=0$  transition (broken line) as they appear superimposed near center frequency for the indicated first side line Zeeman frequency.

#### Conclusions

Although not everything is understood in the results of the measurements presented in this paper, it is clear from fig. 5 that an accurate positioning of the zero Rabi pulling can be achieved in both directions of the beam, resulting in a value for the end-to-end cavity phase shift,  $\delta\phi$ , which appears to have an uncertainty of less than  $\pm$  5 x 10 $^{-14}$ . We will therefore state as our result

$$\frac{\delta v}{v} \mid_{\delta_{\Phi}} = (4.5 \pm 0.5) \times 10^{-13}$$
 (4)

This is quite consistent with previously published recent results for the cavity phase shift of NBS-6 shown in figure 9.

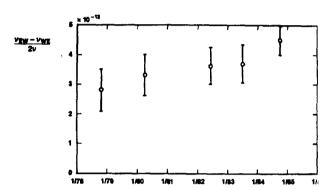


Fig. 9. Recent published results for the cavity phase shift of NBS-6.

The change in the measurements with time has been often interpreted as a drift of the phase difference itself possibly due to unequal cesium deposition. Although this is quite possible we suggest here that care must be taken in drawing conclusions because the Rabi pulling effect has not been adequately accounted for in the past. However, it turns out that all these measurements have been taken at C-field values for which the Rabi pulling is relatively small.

The Rabi pulling is strongly dependent on microwave power at most C-field values and the field values where this dependence vanishes may depend on anything affecting beam optics: from the field strength of selecting magnets (and their demagnetization with time), to source and detector position (with each beam reversal), to changes introduced by major redesign of tube ends as in the modification of NBS-5 into NBS-6. It can be expected that once the effect of all these variables on the zero crossing positions of the Rabi pulling is understood and repeatability can be guaranteed to the necessary degree, the documented accuracy claims for NBS-6 may improve to less than  $10^{-13}$  and possibly to few parts in  $10^{-14}$ , the residual uncertainties being probably , the residual uncertainties being probably due in such a case to electronics and C-field inhomogeneity. The long term stability should also benefit from operation at a zero pulling point because of the small frequency dependence on microwave power.

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### References

- [1] R. L. Lacey, Proc. 22nd Annual Symp. on Freq. Contr. 545-558 (1968).
- [2] A. De Marchi, G. P. Bava, Metrologia 20, 33-36 (1984).
- [3] D. J. Wineland, Metrologia 13, 121-123 (1977).
- [4] S. Jarvis, Jr., D. J. Wineland, H. Hellwig, J. Appl. Phys. 48, 5336-5337 (1977).
- [5] H. Hellwig, J. A. Barnes, D. J. Glaze, Proc 27th Annual Symp. on Freq. Contr., 309-312 (1973).
- [6] H. Hellwig, S. Jarvis, D. Halford, H. E.
- Bell, Metrologia 9, 107-112 (1973). [7] D. A. Howe, H. E. Bell, H. Hellwig, A. De Marchi, Proc 28th Annual Symp. on Freq. Contr., 363-372 (1974).
- S. Jarvis, Jr., NBS Tech. Note 660, NBS (1975).
- [9] D. W. Allan, H. Hellwig, S. Jarvis, Jr., D. A. Howe, R. A. Garvey, Proc. 31st Annual Symp. on Freq. Contr., 555-561 (1977).
- Metrologia 20, 37-67 (1984).
- [12] D. J. Glaze, H. Hellwig, D. W. Allan, S. Jarvis, Jr, Metrologia 13, 19-28
- [13] F. L. Walls, K. B. Persson, Proc. 38th Annual Symp. on Freq. Contr., 416-419 (1984).